The Background of Computation

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Extended Abstract

Backgrounds of various kinds are ubiquitous. A beautifully written book on mathematics may be inaccessible to a person without the proper mathematical background. A person with no scientific background may have hard time distinguishing between scientific and non-scientific arguments. A well intentioned and well behaved person may seem rude in a different country, with a different cultural background.

Computations also have backgrounds. If you program in C, you know that there is a big difference between arrays and, say, multisets. If you need multisets, you have to program them one way or another. But there is no need to program arrays. Arrays are just there, available for free. In our terms, arrays are in the background of C but multisets are not.

Every programming language has a background. But backgrounds are not restricted to programming languages. A software library may be an additional background resource for your program. Backgrounds are often tacitly assumed. We wanted to bring them into the open. In 2000, we introduced and studied a precise and rather general notion of background for computations. The idea was (and still is) to associate to each set S a structure, consisting of "background objects" like arrays, multisets, etc., built over S. Elements of S are treated as atoms. A background is called finitary if each background object depends on only finitely many atoms.

It turned out, however, that the notion was not general enough. In particular, the collection of hereditarily finite sets (over some set of atoms) with the containment relation did not constitute a background according to our definition, but intuitively it should. We present a new, more comprehensive theory of the computation background. Previously, we required that every embedding between two sets extends uniquely to an embedding between the associated background structures. The new, more general theory allows for several such extensions, but it requires one of them to be distinguished as the standard one.

The background elements are constructed from atoms, at least in a weak sense. We also consider various stronger degrees of constructivity of backgrounds.

This is joint work with Andreas Blass.